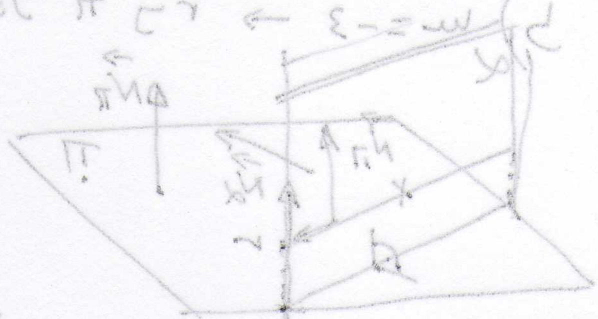
2006/3/A/4.



$$\Pi \equiv 2x+y-z+2=0$$

$$r \equiv \frac{x-5}{-2} = y = \frac{z-6}{m}.$$

a) Pos. relativa de Π y r en función de m.

Pasamos r a f. general: $x-5=-2y$, $my=z-6$ $\quad$ $\left.\begin{array}{l} x+2y=5 \\ my-z=-6 \end{array}\right|$

Así el sist. de ec. formado por Π y r será:

$$\left.\begin{array}{l} 2x+y-z=-2 \\ x+2y=5 \\ my-z=-6 \end{array}\right\} \quad A/A^* = \left(\begin{array}{ccc|c} 2 & 1 & -1 & -2 \\ 1 & 2 & 0 & 5 \\ 0 & m & -1 & -6 \end{array}\right)$$

$\Rightarrow$ rango A: $\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \neq 0 \Rightarrow$ rango $A \geq 2$.

$$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ 0 & m & -1 \end{vmatrix} = -4-m+1 = -3-m.$$

1º Si $-3-m=0 \Rightarrow \boxed{m=-3}$; rango $A=2$. Est. rango A^*:

$$A^* = \left(\begin{array}{ccc|c} 2 & 1 & -1 & -2 \\ 1 & 2 & 0 & 5 \\ 0 & -3 & -1 & -6 \end{array}\right) \to \left[A^* = \left(\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 1 & 2 & 0 & 5 \\ & & & \end{array}\right)\right]$$

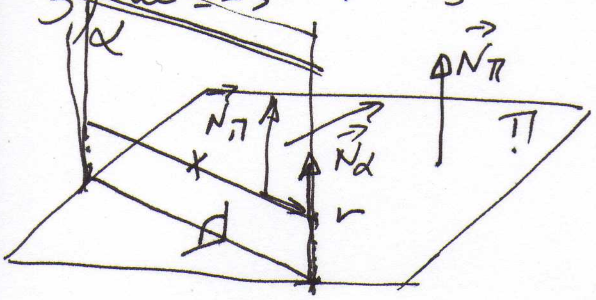
$\boxed{F_3 = F_1 - 2F_2}$ =

rango A^*: $\begin{vmatrix} 2 & 1 & -2 \\ 1 & 2 & 5 \\ 0 & -3 & -6 \end{vmatrix} = -24+6+30+6 \neq 0 \Rightarrow$ rango $A^* = 3$

$\left.\begin{array}{l} \text{rango } A = 2 \\ \text{rg } A^* = 3 \end{array}\right|$ Sist. Incompatible. No sol. $\Rightarrow r$ y Π //.

2º $\forall m \neq -3 \in \mathbb{R}$ $\left.\begin{array}{l} \text{rango } A = 3 \\ \text{rango } A^* = 3 \end{array}\right|$ Sist. Compat. Det. Sol. única r y Π se cortan en un pto.

b) $m=-3 \rightarrow r$ y π son // $\quad r \equiv \frac{x-5}{-2} = \frac{y}{1} = \frac{z-6}{-3}$



$P(5,0,6)$

$\vec{v}_r\ (-2,1,-3)$

$$\alpha \perp \pi \Rightarrow \begin{cases} r \in \alpha \Rightarrow P(5,0,6) \\ \qquad\quad \vec{v}_r(-2,1,-3) \\ \vec{N_\pi} \quad (2,1,-1) \end{cases}$$

$\pi \equiv 2x+y-z+2=0$

$$\alpha \equiv \begin{vmatrix} x-5 & y & z-6 \\ -2 & 1 & -3 \\ 2 & 1 & -1 \end{vmatrix} = -(x-5)-2(z-6)-6y - 2(z-6)+3(x-5)-2y =$$

$= 2x-8y-4z+5+12+12-15 = 2x-8y-4z+14=0$

$(\div 2)\ \boxed{\alpha \equiv x-4y-2z+7=0}$

$(1,-4,-2)$

$\underline{2+1-1}$

$2-4+2=0$